DISCRETE PROBABILITY DISTRIBUTIONS

NAME	EXPERIMENT	PROBABILITY MASS FUNCTION (pmf), $p(x)$	MEAN (μ), VARIANCE (σ^2), MOMENT GENERATING FUNCTION $(\phi(t))$	COMMENTS
Discrete Uniform	Equally likely k different values	$\frac{1}{k},\ x = x_1, x_2, \ldots, x_k$		
Bernoulli	• two possible outcomes $p = P\{success\} = P\{X = 1\}$	$p^x(1-p)^{x-1}$, $x=0,1$	$\mu = p,\ \sigma^2 = p(1-p)$ $\phi(t) = 1 + p(e^t - 1)$	
Binomial	• two possible outcomes • fixed number of trials (n) • $p = P\{success\}$ is fixed from trail to trial • independent trials	X=the number of successes out of n trials $P\{X = x\} = \binom{n}{x} p^x (1-p)^{x-1}$, $x=0,1,\ldots,n$	$\mu = np,\ \sigma^2 = np(1-p)$ $\phi(t) = \left[1 + p(e^t - 1)\right]^n$	• Let $Y = \frac{X}{n}$, then $\mu_Y = p$, $\sigma_Y^2 = \frac{p(1-p)}{n}$
Negative Binomial	• two possible outcomes • no fixed number of trials • $p = P\{success\}$ is fixed from trail to trial • independent trials	X=the number of trials at which the kth success occurs. $P\{X = x\} = \binom{x-1}{k-1} p^x (1-p)^{x-k}$ $x=k,k+1,\ldots$	$\mu = \frac{k}{p},\ \sigma^2 = \frac{k}{p}\left(\frac{1}{p} - 1\right)$ $\phi(t) = \left[\frac{pe^t}{(1-(1-p)e^t)}\right]^k$	• If k=1 then it is called a geometric distribution. This distribution is memoryless. • For the geometric distribution $F(x) = P\{X \le x\} = 1 - (1-p)^x$
Hypergeometric	• N individuals in the population • two possible outcomes M=number of successes in the population • n individuals are selected without replacement	X=the number of successes out of n trials $P(X = x) = h(x; n, N, M)$ $= \frac{\binom{M}{x}\binom{N-M}{n-x}}{\binom{N}{n}}$ for $\max(0, n - N + M) \le x \le \min(n, M)$	$\mu = \frac{nM}{N}$ $\sigma^2 = \frac{nM(N-M)(N-n)}{N^2(N-1)}$	• used when we sample without replacement
Poisson	• counts number of events in one unit • probability that an event occurs in one unit is same for all units • the number of events in units are independent	X=the number of times an event occurs in one unit $P(X = x) = p(x; \lambda)$ $= \frac{\lambda^x e^{-\lambda}}{x!}$ for $x = 0, 1, \ldots$	$\mu = \sigma^2 = \lambda$ $\phi(t) = e^{\lambda(e^t - 1)}$	• Poisson Approximation to Binomial If X has *Bin(n,p)* $P\{X = x\} \cong \frac{e^{-\lambda}\lambda^x}{x!}$ where $\lambda = np$
Multinomial	• k possible outcomes • fixed number of trials (n) • $\theta_i = P(\text{"}ith\ outcome\text{"})$ is fixed from trail to trial • independent trials	X_i=*outcomes of the ith kind*. $P(X_1 = x_1, \ldots, X_k = x_k) = f(x_1, \ldots, x_k; n, \theta_1, \ldots, \theta_k) = \binom{n}{x_1, \ldots, x_k} \theta_1^{x_1} \ldots \theta_k^{x_k}$		
Multivariate Hypergeometric	• N individuals in the population • k possible outcomes M_i=number of kind i in the population • n individuals are selected without replacement	X_i=*outcomes of the ith kind*. $P(X_1 = x_1, \ldots, X_k = x_k) = f(x_1, \ldots, x_k; n, M_1, \ldots, M_k) = \frac{\binom{M_1}{x_1}\binom{M_2}{x_2} \ldots \binom{M_k}{x_k}}{\binom{N}{n}}$		