CONTINUOUS PROBABILITY DISTRIBUTIONS

NAME	PROBABILITY DENSITY FUNCTION (PDF), *f(x)*	MEAN (μ), VARIANCE (σ^2), MOMENT GENERATING FUNCTION $(\phi(t))$	COMMENTS
Uniform over (a, b)	$\begin{cases} \frac{1}{b-a} & a < x < b \\ 0 & otherwise \end{cases}$	$\mu = \frac{a+b}{2}$, $\sigma^2 = \frac{(b-a)^2}{12}$ $\phi(t) = \frac{e^{tb} - e^{ta}}{t(b-a)}$	• $F(x) = P\{X \le x\} = \frac{x-a}{b-a}$ • non-informative, randomness distribution
Gamma parameters (n,λ), $\lambda > 0$	$\begin{cases} \frac{\lambda e^{-\lambda x}(\lambda x)^{n-1}}{(n-1)!} & x > 0 \\ 0 & x \le 0 \end{cases}$	$\mu = \frac{n}{\lambda}$, $\sigma^2 = \frac{n}{\lambda^2}$ $\phi(t) = \left(\frac{\lambda}{\lambda - t}\right)^n$	• Very rich family with different shapes
Exponential parameters (n,λ), $\lambda > 0$	$\begin{cases} \lambda e^{-\lambda x} & x > 0 \\ 0 & x \le 0 \end{cases}$	$\mu = \frac{1}{\lambda}$, $\sigma^2 = \frac{1}{\lambda^2}$ $\phi(t) = \frac{\lambda}{\lambda - t}$	• Gamma with *n=1* • $F(x) = 1 - e^{-\lambda x}$ $P\{X > x\} = e^{-\lambda x}$ (survival function) • memoryless property
Chi-square parameter υ	$= \begin{cases} \frac{x^{(\upsilon/2)-1}e^{-x/2}}{2^{\upsilon/2}\Gamma(\upsilon/2)} & x > 0 \\ 0 & x \le 0 \end{cases}$ υ (nu) is called the degrees of freedom	$\mu = 2\upsilon$, $\sigma^2 = \upsilon$ $\phi(t) = (1-2t)^{-\upsilon/2}$	• Gamma with $\lambda = 1/2$, $n = \upsilon/2$
Beta parameters (α,β)	$f(x) =$ $\begin{cases} \frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)} x^{\alpha-1}(1-x)^{\beta-1} & for\ 0 < x < 1 \\ 0 & otherwise \end{cases}$	$\mu = \frac{\alpha}{\alpha+\beta}$ $\sigma^2 = \frac{\alpha\beta}{(\alpha+\beta)^2(\alpha+\beta+1)}$	• A good model for proportions (Bayesian inference)
Normal parameters (μ,σ)	$n(x;\mu,\sigma) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}$ $for\ -\infty < x < \infty$ **Standard Normal μ=0, σ=1**	$\mu = \mu$, $\sigma^2 = \sigma^2$ $\phi(t) = e^{\mu t + \frac{\sigma^2 t^2}{2}}$	• Bell shaped curve • To find a normal probability use the Table 2.3 on page 81 • If X has $N(\mu,\sigma)$ then $Z = \frac{X-\mu}{\sigma}$ has $N(0,1)$ • **Normal approximation to binomial**. Let X has Binom(n,θ). Make the continuity correction and use the fact that $Z = \frac{X - n\theta}{\sqrt{n\theta(1-\theta)}} \to N(0,1)$
Bivariate Normal	$f(x,y) = \frac{e^{-\frac{1}{2(1-\rho^2)}\left[\left(\frac{x-\mu_1}{\sigma_1}\right)^2 - 2\rho\left(\frac{x-\mu_1}{\sigma_1}\right)\left(\frac{y-\mu_2}{\sigma_2}\right) + \left(\frac{x-\mu_2}{\sigma_2}\right)^2\right]}}{2\pi\sigma_1\sigma_2\sqrt{1-\rho^2}}$ $\mu_1 = E(X)$, σ_1 = Standard deviation of X, $\mu_2 = E(Y)$, σ_2 = Standard deviation of Y, ρ = correlation coefficient between X and Y. **Circular normal distribution ρ=0, σ_1=σ_2**		• If X and Y have a bivariate normal distribution then 1. Y given X=x has a normal distribution with $\mu_{Y\mid x} = \mu_2 + \rho\frac{\sigma_2}{\sigma_1}(x - \mu_1)$ $\sigma^2_{Y\mid x} = \sigma_2^2(1-\rho^2)$ 2. X given Y=y has a normal distribution with $\mu_{X\mid y} = \mu_1 + \rho\frac{\sigma_1}{\sigma_2}(y - \mu_2)$ $\sigma^2_{X\mid y} = \sigma_1^2(1-\rho^2)$ 3. X and Y are independent iff *ρ=0*.