STATISTICAL METHODS

Inquire | Explore | Generalize | Evaluate | Communicate

UNDERSTAND THE UNKNOWN

STAT. 2601: STATISTICAL METHODS
INFORMATION ON THE SECOND MIDTERM EXAMINATION

Date : October 22, 2010 (Friday)
Time: 9:15-10:20
Place: SCI. 3550
Examination Type: Closed notes and books. But you will be allowed to use one sheet of paper (information sheet) with the formulas and facts that you need (This sheet should not have solutions of problems or examples)
The things that you need for the exam:
(1) A calculator with exponential key and/or photocopy of Table V,
(2) Photocopies of Table II, III, and IV
(3) Formula Handout
Coverage: Sections 4.1-6.3 (included)
Previous exams: On the course outline page at www.morris.umn.edu/~sungurea/introstat/index2601.html
Interactive practice test: http://umconnect.umn.edu/p19098565/

The important topics that you should know for the exam.

Chp. 4 Discrete Random Variables

4.1 Types of random variables (Discrete and Continuous)
4.2 Probability distributions for discrete random variables
- Construction of a discrete probability distribution
- Finding probabilities by using a given probability distribution

4.3 Expected values of a discrete random variable
- Finding mean, variance and standard deviation

4.4 The binomial random variable
- Verifying the characteristics of a binomial random variable
- The probability distribution of a binomial random variable
- The mean, variance and standard deviation of a binomial random variable
- Finding probabilities related with the binomial distribution by using (i) Probability distribution p(x), (ii) Binomial Tables

4.5 The Poisson random variable
- Verifying the characteristics of a Poisson random variable
- The probability distribution of a Poisson random variable
- The mean, variance and standard deviation of a Poisson random variable
- Finding probabilities related with the Poisson distribution by using Poisson probability distribution
- Poisson approximation to Binomial probabilities

Chp. 5 Continuous Random Variables

5.1 Continuous probability distribution
5.2 The uniform distribution
- The probability distribution of a uniform random variable on the interval c to d
- The mean and standard deviation of a uniform random variable
- Finding probabilities for the uniform distribution by calculating the areas of the rectangles

5.3 The normal distribution
- Finding probabilities for standard normal and general normal distribution

Given probabilities, finding x, z, quartiles etc.
Use of Normal Table

5.4 Normal approximation to the binomial distribution
Steps 1-4 on pages 215

5.5 The exponential distribution
The probability distribution of an exponential random variable
The mean and standard deviation of an exponential random variable
Finding the Area A to the right of a number a for an exponential distribution

<u>SOLVING PROBLEMS WHICH INVOLVES MORE THAN ONE PROBABILITY DISTRIBUTION</u>

Chp. 6 Sampling Distributions

The difference between parameter and statistics

6.1 Sampling distribution
Finding sampling distribution of a statistics
Finding the mean and standard deviation of a sampling distribution

6.2 Properties of sampling distribution
Point estimator
Unbiased estimate
Biased estimate
Minimum variance estimator

6.3 Central Limit Theorem (CLT)
Sampling distribution of $\bar{x}$
The mean of $\bar{x}$
The standard deviation of $\bar{x}$ = standard error
Approximate distribution of the sample mean for large n (CLT)

EXAM II STUDY QUESTIONS

CHAPTER 4

1. An employee of a firm has an option to invest $1,000 in the company's bonds. At the end of 1 year, the company will buy back the bonds at a price determined by its profits for the year. From the past years, the company predicts it will buy the bonds back at the following prices with the associated probabilities (x = price paid for bonds):

x	$0	$500	$1,000	$1,500	$2,000
p(x)	.01	.22	.30	.22	.25

a. What is the probability the employees will receive $1,000 or less for the investment?
b. What is the expected price paid for the bonds?
c. What is the employee's expected profit?
d. Find the variance and standard deviation for this probability distribution.

2. In a poll conducted by Parents magazine, 60% of parents said they wished they had received more education (Parents, August 1988). A random sample of twenty parents is selected.
a. What is the probability distribution of x, the number of parents who hold this view in this sample? Justify your answer.
b. What is the expected number of parents who will hold this view in this sample? What is the standard deviation of the random variable in part (a)?
c. Find the probability that exactly 3 **<u>will not</u>** hold this view in the sample.
d. Use the normal approximation to find $P(6<x<9)$.

3. A particular type of birth-control pill is 90% effective. A random sample of 20 persons is selected.

a. What is the probability distribution of x, the number of unplanned births for this sample? Justify your answer?
b. How many unplanned births would you expect in this sample? What is the standard deviation of the random variable defined in part (a)?
c. Find the probability that exactly 3 out of 20 will have unplanned births?

4. Consider writing onto a computer disk and sending it through a certifier that counts the number of missing pulses. Suppose this number X has a Poisson distribution with mean 0.2. (Suggested in "Average Sample Number for Semi-Curtailed Sampling Using Poisson Distribution," *J. Quality Technology*, 1993**)**.
a. What is the probability that a disk has exactly one missing pulse?
b. What is the probability that a disk has at least two missing pulses?

5. For a recent period of 100 years, there were 93 major earthquakes (at least 6.0 on the Richter scale) in the world (based on data from the *World Almanac and Book of Facts*). Assuming that the Poisson distribution is a suitable model,
a. Find the mean number of major earthquakes per year and the standard deviation.
b. Find the probability that the number of major earthquakes in a randomly selected year is 5.
c. Find the probability that the number of major earthquakes in a randomly selected year is at least 2.

CHAPTER 5 & 6

6. The speeds of all cars traveling on a stretch of Interstate Highway I-95 are normally distributed with a mean of 68 miles per hour and a standard deviation of 3 miles.
a. Find the percentage of travelers who are violating the 65 miles speed limit.
b. If a police officer decides to give a ticket to the fastest 10% of the drivers, what should be the minimum speed he would use to write a speeding ticket?
c. If a random sample of 36 cars traveling on this highway has been selected, what is the probability that their average speed will exceed the 65 miles speed limit?

7. Based on the sample data collected in the Denver area, Nicholas Kiefer (1985) found that in some cases the exponential distribution is an adequate approximation for the distribution of the time (in weeks) an individual is unemployed. Use $\theta = 13$ to answer the following questions.
a. What is the mean time workers are unemployed according to the exponential distribution?
b. Find the probability that a worker who just lost her job will be unemployed for at least two weeks.
c. What is the probability that an unemployed worker will find a new job within 12 weeks?
d. What is the probability that only two of the five unemployed workers will find a new job within 12 weeks?

8. In 1987, the average annual rate of interest paid by savings and loan institutions in Pennsylvania was 7.26%. Assume a normal distribution and a standard deviation of 1.50% to answer the following questions.
a. What is the probability that a randomly selected Pennsylvania savings and loan institution paid between 7.00% and 8.00% interest on deposits?
b. What is the probability that a randomly selected 9 such institutions on the average paid between 7.00% and 8.00% interest on deposits?
c. What is the probability that only 5 of the 9 such institutions paid between 7.00% and 8.00% interest on deposits?

9. In deciding how many customer service representatives to hire an in planning their schedules, it is important for a firm marketing electronic typewriters to study the repair times for the machines. Such a study revealed that repair times have an exponential distribution with θ=22 minutes
a. What is the expected repair time for an electronic typewriter?
b. Find the probability that a repair time will last less than 10 minutes.
c. The charge for typewrite repair is $50 for each half hour, or part thereof, for labor. What is the

probability that a repair job will result in a charge for labor of $100?

10. The board of examiners that administered the real estate brokers' examination in a certain state found that the mean score on the test was 435 and the standard deviation was 72. If the board wants to set the passing score so that only the best 30% of all applicants pass, what is the passing score? Assume that the scores are normally distributed.

11. A taxi service based at an airport can be characterized as a transportation system with one source terminal and a fleet of vehicles that take passengers from the terminal to different designations. Each vehicle returns to the terminal after some random trip time and makes another trip. To improve the vehicle-dispatching decisions involved in such system, Sims and Templeton (1985) assumed travel times of successive trips are independent exponential random variables to model the system. Assume $\theta = 20$ minutes.

a. What is the mean trip time for the taxi service?

b. What is the probability that a particular trip will take less than 30 minutes.

c. What is the probability that a particular trip will take between 10 to 15 minutes.

d. Two taxis have just been dispatched. What is the probability that both will be gone for more than 30 minutes? That at least one of the taxis will return within 30 minutes?

CHAPTER 6

12. A state game protector collects measurements on the weights of species of fish in a lake. Suppose that the weight has a normal distribution with mean 7.34 and a standard deviation of 2.13 pounds. A total of 49 measurements are obtained.

a. What is the sampling distribution of the average weight of 49 fish?

b. What is the probability that a randomly selected fish will weigh more that 7.5 pounds?

c. What is the probability that the average weight of 49 fishes will exceed 7.5 pounds?

d. Suppose that the weight of fish has a mean 7.34 and standard deviation 2.13. But the distribution of the weight is unknown. What are your answers to **a**, **b**, and **c**.