# 5 Part II

## Bayesian Inference for Continuous Random Variables

### 1. BAYESIAN INFERENCE

The posterior distribution of the parameter given the data is the entire inference from a Bayesian point of view. Therefore, finding the posterior distribution by using the following Bayes' theorem is enough for the inference

***Posterior = (likelihood x prior)/marginal*.**

***Prior*** is the distribution of the prior that has been set up at the beginning.

***Likelihood*** is the conditional distribution of the data given the parameter.

***Marginal*** is the distribution of data.

***Posterior*** is the conditional distribution of the parameter given the data.

In frequentist approach statistical inference involves;

- Point estimation,
- Interval estimation, and
- Hypotheses Testing.

In Bayesian setting ***point estimate*** is any measure of center for the posterior distribution, commonly the ***mean***.

The interval estimate is the ***credibility/credible*** interval is a range of values within which unobserved parameter value will fall after observing data under the specified prior. It is an interval in the domain of the posterior distribution with attached probability. In other words, a $(1-\alpha) \times 100\%$ Bayesian credible interval for $\theta$ is a range of parameter values that has posterior probability $(1-\alpha)$.

We can test a ***one-sided hypothesis*** in a Bayesian manner by computing the posterior probability of the null hypothesis. This probability is found by integrating the posterior density over the null region. If this probability is less than the level of significance a, then we reject the null hypothesis.

We cannot test a ***two-sided hypothesis*** by integrating the posterior probability over the null region because, the prior probability of a point null hypothesis is zero, so the posterior probability will also be zero. Instead, we test the credibility of the null value by observing whether or not it lies within the Bayesian credible interval. If it does, the null value remains credible and we can't reject it.

If the prior and posterior is a member of the same family such as Normal, Beta etc. with different parameters then the prior is called ***conjugate prior***. If you have no knowledge about the parameter at all, you can use the uniform prior which gives equal weight to all values.

## 2. BAYESIAN INFERENCE ON PROPORTIONS/BINOMIAL WITH CONTINUOUS PRIOR

***Conjugate prior*** in this case is a Beta distribution. This distribution has two parameters *a* and *b* and can only take on values between 0 and 1. The mean of the Beta(a,b) is a/(a+b), and the variance is $ab/[(a+b)^2(a+b+1)]$. When $a$=1, $b$=1 we end up with the ***Uniform distribution*** on the interval 0 to 1.

When a<b, the density has more weight in the lower half. The opposite is true when a>b. When a=b, Beta(a,b) density is symmetric. When a=1/2 much more weight is given to values near 0, and when b=1/2 much more weight is given near 1.

The main result is, if the prior is Beta(a,b), then the posterior will be Beta(a+y,b+n-y), where y is the number of successes out of n trials.

## 3. USING R FOR BAYESIAN INFERENCE ON PROPORTIONS/BINOMIAL WITH BETA PRIOR

To carry out the Bayesian analysis on Binomial for the Beta prior by using R we need a package Bolstad.

**library(Bolstad)**

The functions name is binobp(). The syntax for binobp is

binobp(y , n, a = 1, b = 1).

The function takes four values. However the author has specified default values for a, b, namely a = 1 and b = 1. This means that the user only has to supply the argument y, number of successes, and n, number of trials. Hence the simplest example of binobp is given as

binobp(6,8)

If the user wants to change the prior used, say to Beta(5,6), then they would type

binobp(6, 8, 5, 6)